**The Plane**

A plane is a surface such that if any two points on it are taken, a straight line joining them lies wholly on the surface, i.e. every point of the line will be on the plane.

* The general equation of the first degree in , and (), always represents a plane. Here, , and are the d’rs of the plane (i.e. the d’rs of the normal to the plane) and is any constant.
* The equation of a plane which passes through the origin is .
* The equation of a plane which passes through any point is .
* The equation of a plane which meets to co-ordinate axes is . Thus, , and .
* The equation of a plane through the intersection of the places and is , where is any constant.
* The angle between two planes is the angle between the normal of the planes.

and

Let be the angle between the two planes.

, , are the d’rs of normal and , and are the d’rs of normal .

and , ,

10. a Find the equation of the plane through the intersection of the planes and and the point .

The equation of the plane through the intersection of the planes and is - (i)

Since this equation passes through the point ,

Putting the value of in (i),

This is the required equation of the plane.

10.b. Find the equation of the plane through the point and perpendicular to each of the planes and .

The equation of the plane through the point is

- (i)

Since this equation is perpendicular to the given planes and , we have

Solving this by cross multiplication,

Putting this in (i),

This is the required equation of the plane.

10.c Find the equation of the plane through the point and parallel to the plane .

The equation of the plane through the point is

- (i)

Since (i) is parallel to the given plane ,

Putting these in (i),

12. Find the equation of the plane through the points and and perpendicular to the plane .

The equation of the plane through the point is

- (i)

Since (i) passes through the point ,

- (ii)

Since (ii) is perpendicular to , we have

- (iii)

========================== This sum is unfinished. ==========================

12. a. Find the equation of the plane perpendicular to each of the planes and and at a distance unity from the origin.

Let - (i) be any plane where , and are the d’rs of the normal to the plane.

Since (i) is perpendicular to and ,

and

Also,

Putting these in (i),

Exercise 17

Prove that the points and are equidistant from the plane , and are on opposite sides of it.

======================== Skipping first part of sum. ========================

Putting the point in the left-hand side of (i), we have

(negative value)

Putting the point in the left-hand side of (i), we have

(positive value)

Since the two results have opposite signs, the two points lie on opposite sides of the given plane.

Exercise 30

A variable plane is at a constant distance from the origin and meets the axes in , and . Show that the locus of the centroid of the tetrahedron is .

Let - (i) be the plane which meets the axes in , and where , and .

is the perpendicular to the plane (i).

- (ii)

Let be the centroid of the tetrahedron .

Putting these in (ii),

Hence, the required locus of is .

Exercise 12

A variable plane is at a distance from the origin and meets the axes in , , . Through , , , planes are drawn, parallel to the coordinate planes. Show that the locus of their points of intersection are .

Let - (i) be the plane which meets …

======================== Skipping first part of sum. ========================

- (ii)

The coordinate planes are ( plane), ( plane) and ( plane). Any plane parallel to the plane is . Since the plane passes through , .

Similarly, the equations of the planes parallel to the and planes are and .

So, we have and .

From (ii), the required locus of , , is .

Find the area of the triangle with vertices , and .

Area of the triangle,

Find the volume of the tetrahedron whose 4 vertices are , , and .

Find the volume of the tetrahedron whose 4 faces are

- (i)

- (ii)

- (iii)

- (iv)

Method 1:

Solving (i), (ii) and (iii) we have a point .

Solving (i), (iii) and (iv) we have a point .

Solving (ii), (iii) and (iv) we have a point .

Method 2:

where

and , , and are cofactors of , , and respectively.